A Simple Analysis of a Symmetric Waveguide Five-Port Junction

Marek E. Bialkowski

Abstract—**A simple analysis of a symmetric waveguide five-port junction is described. Based on this analysis a computer algorithm for determining the admittance matrix parameters for the junction is developed. The validity of the algorithm is verified through the comparison with computationally more involved algorithms and experiment.**

I. Introduction

IN recent years, attention has been focused on a symmetric five-port junction as its properties lead to a number of useful applications in microwave engineering [1]–[5]. One particular application is in six-port measurement techniques where a symmetric five-port is used to build an optimal six-port reflectometer [1]–[4].

One of the symmetric five-ports that has been considered so far is created by a junction between a radial cavity and five rectangular waveguides [3], [4]. These rectangular waveguides have their larger sides parallel to the cavity axis.

The theoretical analysis of this five-port for the case when the waveguide widths are equal to the cavity height has been carried out in [3], [4]. By using the least-squares boundary residual method (LSBRM) an accurate model for this junction has been obtained. The drawback of this method is that it is computationally involved.

In this letter, a simplified analysis of the same junction is carried out. In the present analysis, waveguide widths do not have to be equal to the cavity height. The analysis produces closed form expressions for the admittance matrix of the junction.

II. Analysis

The five-port analyzed is shown in Fig. 1. The device is formed by a radial cavity and five identical rectangular waveguides, their larger sides parallel to the cavity axis and separated by an angle of 72 degrees.

Both the cavity and the waveguides are assumed to be formed by a perfect conductor. The cavity may or may not include a concentric conducting post.

It is assumed that at a given frequency the waveguides support only propagation of the dominant TE1,0 mode. The higher order modes are present at the junction but they decay rapidly with a distance along the waveguides.

Manuscript received September 13, 1991.
The author is with the Department of Electrical Engineering, University of Queensland, Queensland 4072, Australia.
IEEE Log Number 9105534.

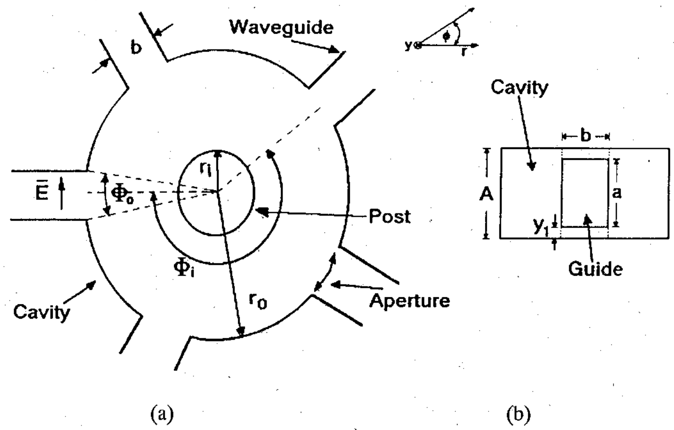

Fig. 1 A cross-section of a symmetric E-plane waveguide five-port junction.

To simplify the analysis it is assumed that the cavity radius r_0 is large in comparison with the waveguides height b, so that waveguide apertures can be considered flat or circular in shape, interchangeably.

Under this assumption the five-port can be characterized in terms of the admittance matrix parameters. In order to determine these parameters one of the apertures needs to be excited, while the remaining ones are short-circuited. In further analysis it is assumed that aperture no. 1 is excited.

The electric field in aperture no. 1 is approximated by the dominant $TE_{1,0}$ rectangular waveguide mode so that its tangential component is given by

$$\overline{E}_t \cong \overline{E}_\phi = \overline{a}_\phi \begin{cases} \sin\frac{\pi}{a}(y-y_1), & \text{for} \begin{cases} y_1 \le y \le y_1 + a, \\ \phi_1 - \frac{\phi_o}{2} \le \phi \le \phi_1 + \frac{\phi_o}{2}, \end{cases} \\ 0, & \text{elsewhere.} \end{cases} \quad (1)$$

Note, that in (1), a planar waveguide aperture was replaced by a circular, cylindrical aperture.

For the excitation (1), the admittance matrix parameters of the five-port can be determined by using

$$y_{i1} = Z_f \frac{2}{a}\frac{1}{\phi_o} \int_{\phi_i-\phi_o/2}^{\phi_i+\phi_o/2} \int_{y_1}^{y_{1+a}} H_y(r=r_0)\, \sin\frac{\pi}{a}(y-y_1)dy\, d\phi, \quad (2)$$

where Z_f is the wave impedance for the $TE_{1,0}$ mode and Hy is the y-component of the magnetic field in the ith aperture.

From (2) it is seen that in order to find $y_{i,1}$, the magnetic field needs to be first determined.

The determination of the magnetic field can be accomplished as follows.

For the excitation (1), the electromagnetic field in the cavity is given in terms of TE (to the y-direction) radial modes. The y-component of the magnetic field can be shown to be described by [3], [4]:

$$H_y(r) = \sum_{n=1}^{N}\sum_{l=-L}^{L} F_{nl}\frac{2}{A}\frac{-j\Gamma_n}{kZ_o}R_{nl}(r)\cdot\sin(k_{yn}y)e^{jl\phi}, \qquad (3)$$

where function R_{nl} is defined by

$$R_{nl}(r) = \begin{cases} \dfrac{J_1(\Gamma_n r)}{J_1'(\Gamma_n \Gamma_o)}, & \text{for a radial cavity,} \\ \dfrac{J_1(\Gamma_n r)Y_1'(\Gamma_n r_i) - Y_1(\Gamma_n r)J_1'(\Gamma_n r_i)}{J_1'(\Gamma_n r_o)Y_1'(\Gamma_n r_i) - Y_1'(\Gamma_n r_o)J_1'(\Gamma_n r_i)}, & \text{for a coaxial cavity,} \end{cases} \qquad (4)$$

where k is the wave number and Zo is the intrinsic impedance for the cavity region, J_l, Y_l are Bessel and Neumann functions of the lth order, $k_{yn} = \frac{n\pi}{A}, \Gamma_n^2 = k^2 - k_{yn}^2, j = \sqrt{-1}$ symbol "$'$" implies a derivative, and F_{nl} are unknown expansion coefficients.

Expansion coefficients F_{nl} can be determined by applying the Fourier analysis to the ϕ-component of the electric field, which at $r = r_0$ is given by

$$E_\phi(r = r_o) = \sum_{n=1}^{N}\sum_{l=-L}^{L} F_{nl}\frac{2}{A}\sin(k_{yn}y)e^{jl\phi}, \qquad (5)$$

where the left-hand side is given by (1).

By multiplying both sides of (5) by $\exp(-jp\phi)$ and by integrading within the limits of $0 < \phi < 2\pi$ unknown expansion coefficients F_{nl} can easily be determined and are given by

$$F_{nl} = \frac{1}{2\pi}\int_{-\phi_o/2}^{\phi_o/2} e^{-jl\phi}\,d\phi \int_{y_1}^{y_1+a} \sin\left(\frac{\pi}{a}(y - y_1)\right)\sin(k_{yn}y)\,dy. \qquad (6)$$

By substituting (3) and (6) into (2), an explicit expression for the admittance matrix parameters can be obtained:

$$y_{il} = \frac{-j}{k}\frac{Z_f}{Z_o}\frac{8}{aA}\frac{\pi}{\phi_o}\sum_{n=1}^{N}\sum_{l=0}^{L} F_{nl}^2\epsilon_{01}\cos(l\phi i)R_{nl}(r = r_o), \qquad (7)$$

where ϵ_{ol} is the Neumann factor.

III. Results

Based on (7), a computer algorithm for determining the admittance matrix for an E-plane waveguide five-port was developed. For low orders l, special functions in (7) were evaluated by using polynomial expansions and recursive formulas given in [6]. For high orders when arguments become small in comparison with l, one term polynomial expansions valid for points close to the origin were used.

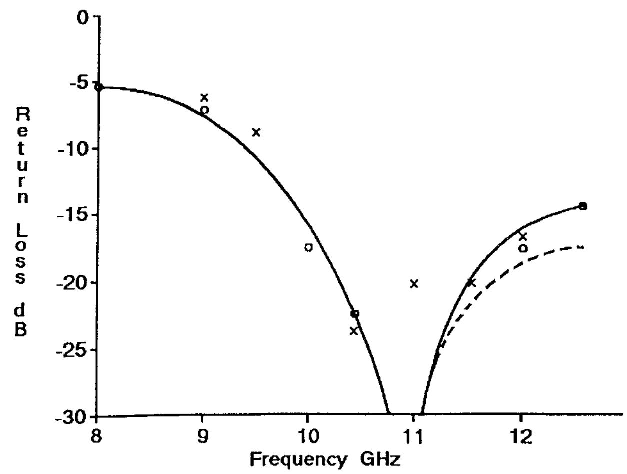

Fig. 2. Return loss versus frequency for a symmetrical five-port waveguide junction. Junction dimensions: $a = 22.86$ mm, $b = 10.16$ mm, $r_o = 13.4$ mm, $r_i = 1.80$ mm, A = a.

To include the case when waveguides are arbitrarily terminated, an additional part of the algorithm based on the circuit theory was written.

The validity of the algorithm could be verified by using the results presented in [3], [4]. Fig. 2 shows the comparison between the numerical results for the return loss of a matched symmetrical five-port obtained with the new algorithm, those obtained by using the least-squares residual method, and experiment [3].

It can be seen that the new algorithm has a reasonably good agreement with more sophisticated theory and experiment presented in [3].

A discrepancy can be observed at higher frequencies. This occurs from the approximation of the electric field in the waveguide aperture by the waveguide TE1,0 mode and from the approximation of planar apertures by circular, cylindrical apertures. Generally, the presented method gives a good qualitative measure of the operation of the five-port junction throughout the entire frequency band of the dominant mode operation of waveguides.

IV. Conclusion

A simple analysis of a symmetric waveguide five-port has been presented. Based on this analysis a computer algorithm for determining the admittance matrix of the five-port has been developed. The algorithm has been verified and shows reasonably good agreement with computationally more involved theory and experiment.

Because of its simplicity the new algorithm has a potential to be extended for more complicated five-ports that would include layered dielectrics.

References

[1] E. R. Bareli Hanson and G. P. Ripely, "An ideal six-port network consisting of a matched reciprocal five-port and a perfect directional

coupler," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 284–288, 1983.

[2] G. P. Ripely, "Matched waveguide five-port with partial symmetry for use in six-port measurement systems," *IEE Proc.*, pt. H, vol. 135, no. 1, pp. 13–16, Feb. 1988.

[3] A. L. Cullen and S. P. Yeo, "Using the least-squares boundary residual method to model the symmetrical five-port waveguide junction," *IEE Proc.*, pt. H, vol. 134, no. 2, pp. 116–124, Apr. 1987.

[4] A. L. Cullen and S. P. Yeo, "Designing a symmetrical five-port waveguide junction," *IEE Proc.*, pt. H, vol. 135, no. 1, pp. 17–22, Feb. 1988.

[5] K. Chang *et al.*, "High-power four-way power divider/combiner," in *IEEE MTT-S Int. Symp., Dig. of Papers*, Dallas, TX, pp. 1329–1332, 1990.

[6] M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions.* New York: Dover Publications, 1965.